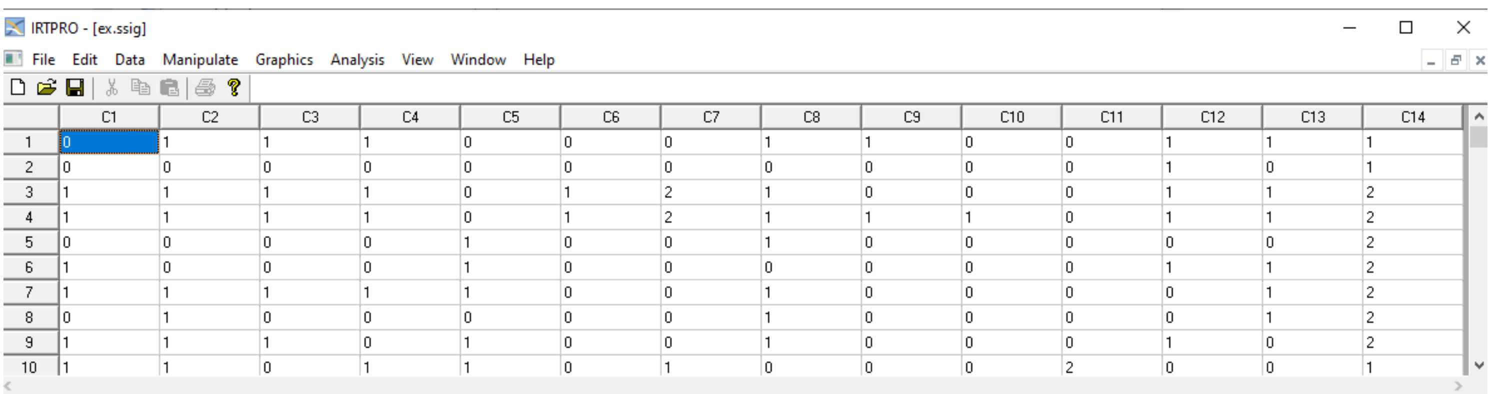
Using a mixture of free and fixed item parameters

Contents

1. Introduction

In this example we illustrate how to set up an IRTPRO™ analysis with a mixture of models and a mixture of free and fixed parameters.

Our example data set **ex.ssig** contains 14 items as shown below. Clearly, not all items are binary in nature. We intend to fit a 3PL model to items 1 through 6, 8, 9, 12 and 13. A graded model is fitted to the remainder, *i.e.*, items 7, 10, 11, and 14.



	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
1	0	1	1	1	0	0	0	1	1	0	0	1	1	1
2	0	0	0	0	0	0	0	0	0	0	0	1	0	1
3	1	1	1	1	0	1	2	1	0	0	0	1	1	2
4	1	1	1	1	0	1	2	1	1	1	0	1	1	2
5	0	0	0	0	1	0	0	1	0	0	0	0	0	2
6	1	0	0	0	1	0	0	0	0	0	0	1	1	2
7	1	1	1	1	1	0	0	1	0	0	0	0	1	2
8	0	1	0	0	0	0	0	1	0	0	0	0	1	2
9	1	1	1	0	1	0	0	1	0	0	0	1	0	2
10	1	1	0	1	1	0	1	0	0	0	2	0	0	1

In the next two sections, we show how to construct the syntax for this model in two ways: by writing the syntax directly, and by creating syntax via the GUI.

1. Writing syntax

Defining the model for the individual items is done via the Model statement. In the Group paragraph below, the 3PL and Graded models are specified for the respective items. By default, IRTPRO™ assumes a 2PL model for binary items, so to obtain 3PL models instead, this paragraph needs to be edited.

```
Group :
    Dimension = 1;
    Items = C1 - C14;
    Codes(C1 - C6) = 0(0), 1(1);
    Codes(C7) = 0(0), 1(1), 2(2), 3(3);
    Codes(C8) = 0(0), 1(1);
    Codes(C9) = 0(0), 1(1);
    Codes(C10) = 0(0), 1(1), 2(2);
    Codes(C11) = 0(0), 1(1), 2(2);
    Codes(C12) = 0(0), 1(1);
    Codes(C13) = 0(0), 1(1);
    Codes(C14) = 0(0), 1(1), 2(2);
    Model(C1 - C6) = 3PL;
    Model(C7) = Graded;
    Model(C8) = 3PL;
    Model(C9) = 3PL;
    Model(C10) = Graded;
    Model(C11) = Graded;
    Model(C12) = 3PL;
    Model(C13) = 3PL;
    Model(C14) = Graded;
    Mean = 0.0;
    Covariance = 1.0;
```

In this single-group analysis demonstration, the population/prior mean is assumed to be zero and the variance is assumed to be 1. See Section 4 for additional comments regarding linking and equating issues under non-equivalent groups.

To fix items to specific values, the Constraints paragraph is used. A 3PL model is fitted to the first item, for example, and values for all three parameters are entered in the format

```
(Item, Slope[0]) = value;
(Item, Intercept[0]) = value;
(Item, Guessing[0]) = value;
```

Note that it is necessary for all three constraints to be added to the syntax – constraints are added only for the specific item parameter the user wishes to constrain.

```
Constraints:
    (C1, Slope[0]) = 0.98606;
    (C1, Intercept[0]) = 0.76990;
    (C1, Guessing[0]) = -1.52905;
    (C2, Slope[0]) = 1.70038;
    (C2, Intercept[0]) = 2.12004;
    (C2, Guessing[0]) = -1.60488;
    (C3, Slope[0]) = 1.14757;
    (C3, Intercept[0]) = -0.82169;
    (C3, Guessing[0]) = -2.52556;
    (C4, Slope[0]) = 2.28316;
    (C4, Intercept[0]) = 1.02720;
```

```
    (C4, Guessing[0]) = -2.24850;
    (C5, Slope[0]) = 0.72650;
    (C5, Intercept[0]) = 0.06501;
    (C5, Guessing[0]) = -1.43274;
    (C7, Slope[0]) = 2.41234;
    (C7, Intercept[0]) = -0.81696;
    (C7, Intercept[1]) = -3.09090;
    (C7, Intercept[2]) = -4.68036;
    (C10, Slope[0]) = 2.60147;
    (C10, Intercept[0]) = -2.12192;
    (C10, Intercept[1]) = -4.05966;
```

In this example we opted to fix the intercept for the 7th item, to which a graded model is to be fitted, using similar syntax of the form:

```
 (C7, Intercept[0]) = -0.81696;
 (C7, Intercept[1]) = -3.09090;
 (C7, Intercept[2]) = -4.68036;
```

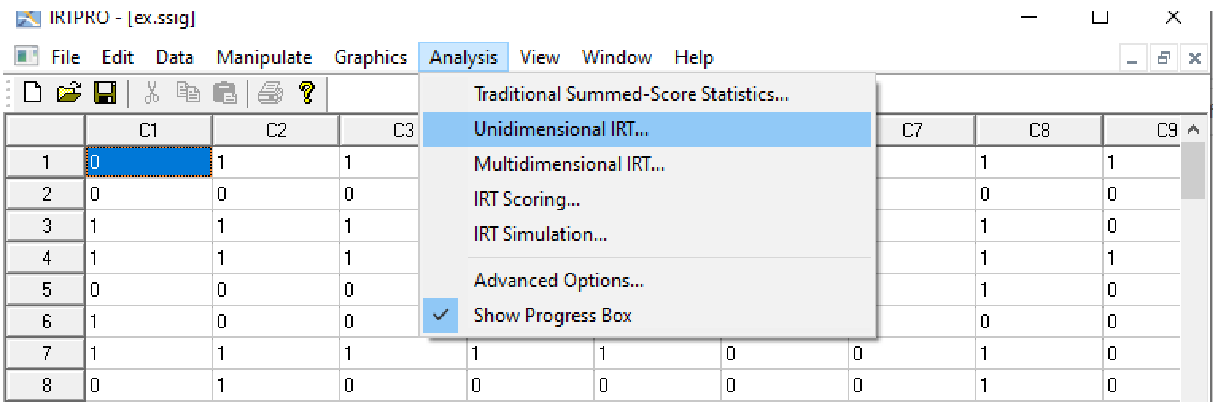
Three statements are entered, corresponding to the three categories of this item (see the Group paragraph previously shown for details). The complete syntax file **ex.irtpro** is shown below.

```
  File = .\ex.ssig;

Analysis:
    Name = Test1;
    Mode = Calibration;

Title:
Comments:
Estimation:
    Method = BAEM;
    E-Step = 500, 1e-05;
    SE = S-EM;
    M-Step = 50, 1e-06;
    Quadrature = 49, 6;
    SEM = 0.001;
    SS = 1e-05;

Scoring:
    Mean = 0;
    SD = 1;

Miscellaneous:
    Decimal = 2;
    Processors = 10;
    Print M2, CTLD, P-Nums, Diagnostic;
    Min Exp = 1;

Groups:

Group :
    Dimension = 1;
    Items = C1 - C14;
    Codes(C1 - C6) = 0(0), 1(1);
```

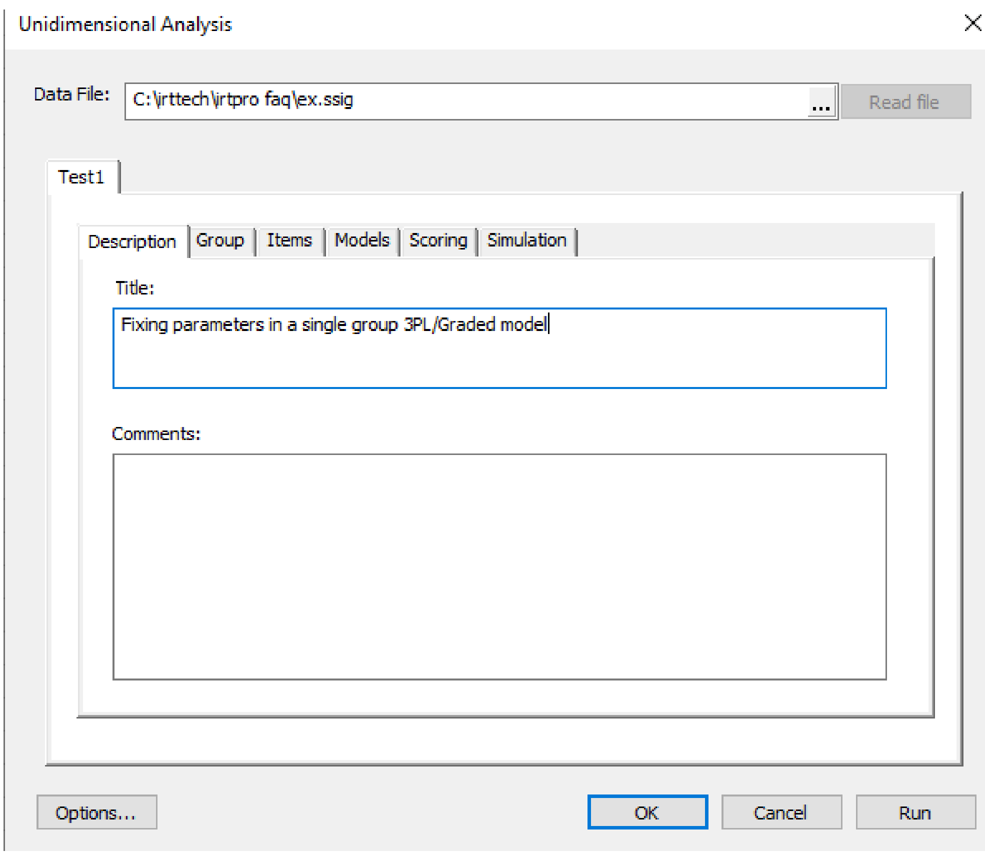
```
    Codes(C7) = 0(0), 1(1), 2(2), 3(3);
    Codes(C8) = 0(0), 1(1);
    Codes(C9) = 0(0), 1(1);
    Codes(C10) = 0(0), 1(1), 2(2);
    Codes(C11) = 0(0), 1(1), 2(2);
    Codes(C12) = 0(0), 1(1);
    Codes(C13) = 0(0), 1(1);
    Codes(C14) = 0(0), 1(1), 2(2);
    Model(C1 - C6) = 3PL;
    Model(C7) = Graded;
    Model(C8) = 3PL;
    Model(C9) = 3PL;
    Model(C10) = Graded;
    Model(C11) = Graded;
    Model(C12) = 3PL;
    Model(C13) = 3PL;
    Model(C14) = Graded;
    Mean = 0.0;
    Covariance = 1.0;

Constraints:
    (C1, Slope[0]) = 0.98606;
    (C1, Intercept[0]) = 0.76990;
    (C1, Guessing[0]) = -1.52905;
    (C2, Slope[0]) = 1.70038;
    (C2, Intercept[0]) = 2.12004;
    (C2, Guessing[0]) = -1.60488;
    (C3, Slope[0]) = 1.14757;
    (C3, Intercept[0]) = -0.82169;
    (C3, Guessing[0]) = -2.52556;
    (C4, Slope[0]) = 2.28316;
    (C4, Intercept[0]) = 1.02720;
    (C4, Guessing[0]) = -2.24850;
    (C5, Slope[0]) = 0.72650;
    (C5, Intercept[0]) = 0.06501;
    (C5, Guessing[0]) = -1.43274;
    (C7, Slope[0]) = 2.41234;
    (C7, Intercept[0]) = -0.81696;
    (C7, Intercept[1]) = -3.09090;
    (C7, Intercept[2]) = -4.68036;
    (C10, Slope[0]) = 2.60147;
    (C10, Intercept[0]) = -2.12192;
    (C10, Intercept[1]) = -4.05966;

Priors:
    (C6, Slope[0]) = Lognormal, 0, 1;
    (C6, Guessing[0]) = Beta, 4, 16;
    (C8, Slope[0]) = Lognormal, 0, 1;
    (C8, Guessing[0]) = Beta, 4, 16;
    (C9, Slope[0]) = Lognormal, 0, 1;
    (C9, Guessing[0]) = Beta, 4, 16;
    (C12, Slope[0]) = Lognormal, 0, 1;
    (C12, Guessing[0]) = Beta, 4, 16;
    (C13, Slope[0]) = Lognormal, 0, 1;
    (C13, Guessing[0]) = Beta, 4, 16;
```

2. Creating syntax via the GUI

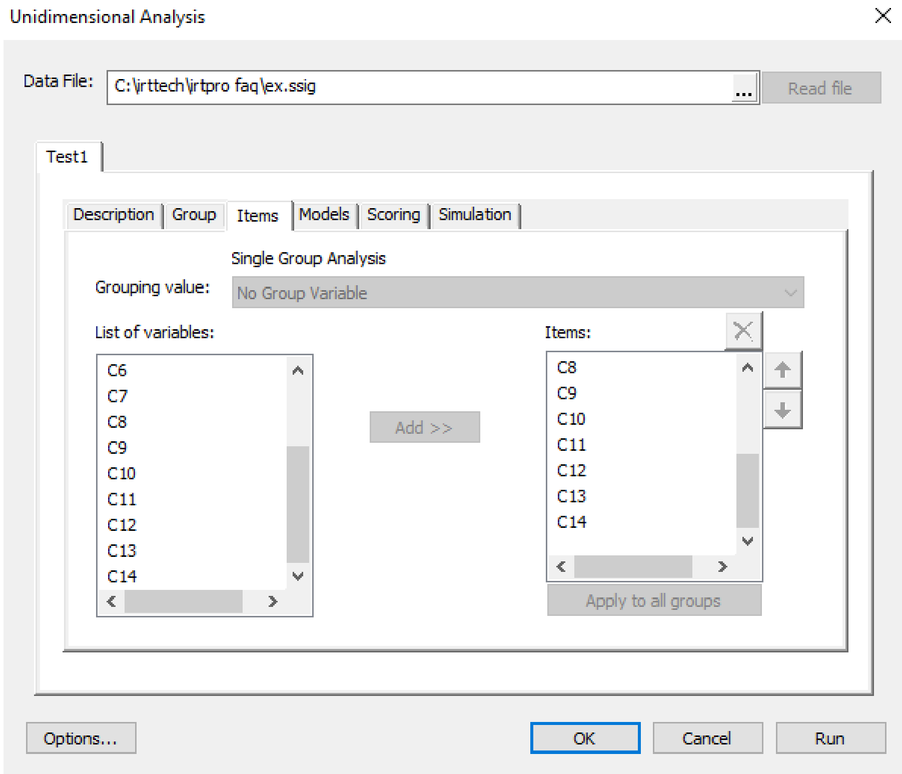
Start by opening the ***.ssig** file. To set up the unidimensional single group analysis, select **Unidimensional IRT** from the **Analysis** menu:



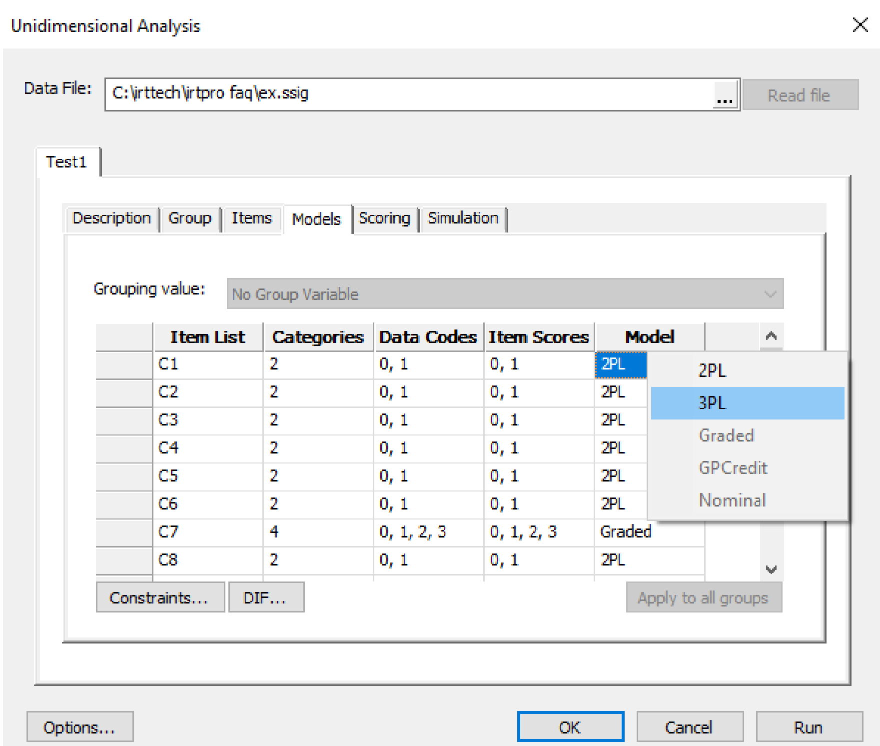
When the **Unidimensional Analysis** dialog box opens, provide a title for the analysis (optional). As this is a single-group analysis, proceed directly to the **Items** tab.



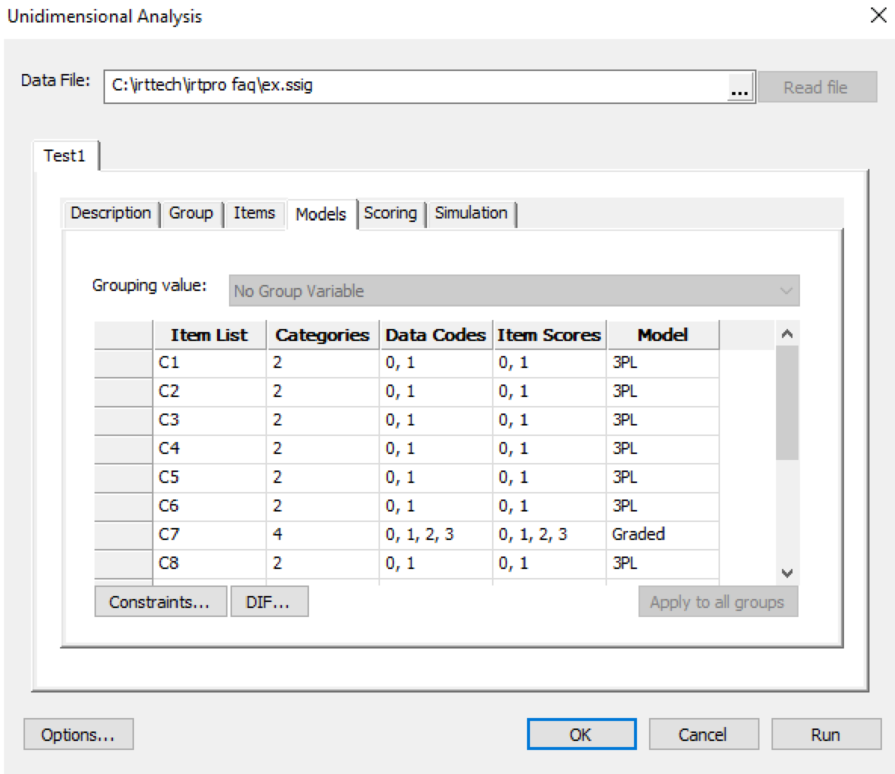
Select all 14 items from the **List of variables** box and click **Add** to add them to the **Items** box. Then click **Models** to access the **Models** tab.



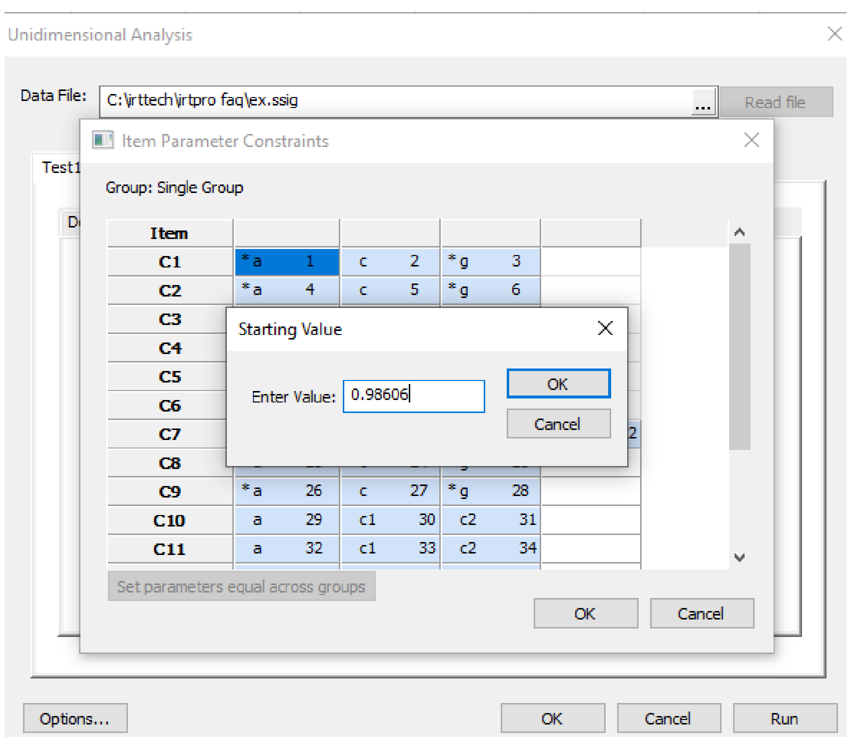
IRTPRO™ automatically identifies non-binary items. By default, it assumes a 2PL model for binary items and a Graded model for non-binary items. To include a guessing parameter for a binary item, right-click in the **Model** field of the item and select 3PL from the pop-up menu as shown below.



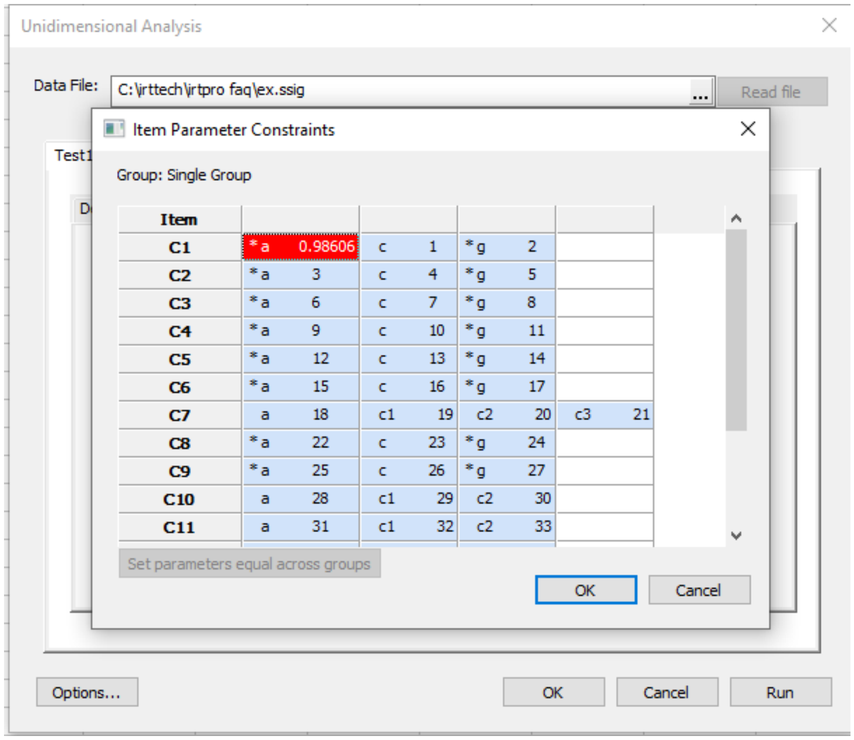
Once items 1 through 6, 8, 9, 12 and 13 have been set to **3PL** instead of the default 2PL model, click the **Constraints** button.



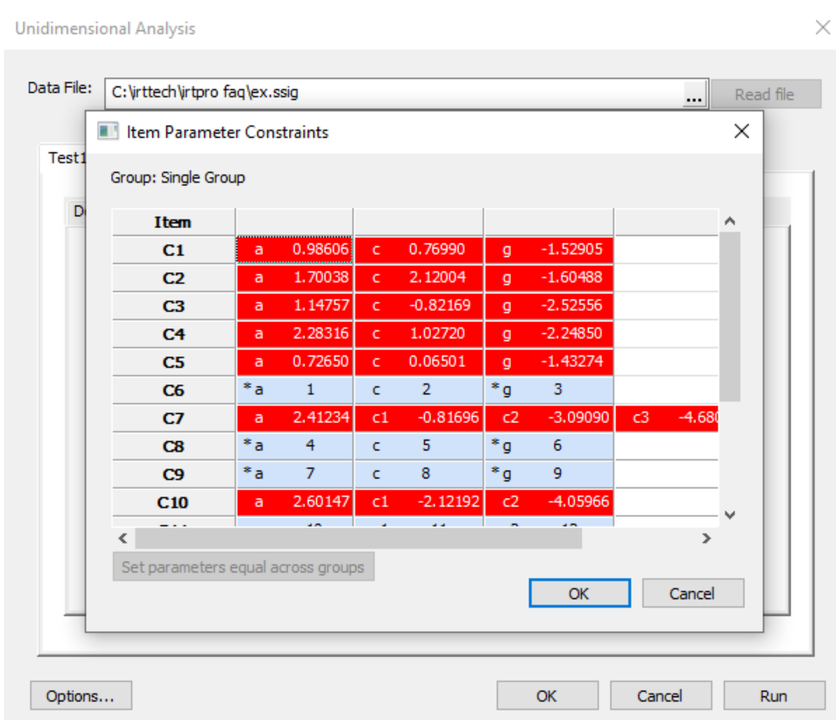
By right-clicking on any field in the **Item Parameter Constraints** dialog box, parameters may be freed, fixed or set equal to one another. To constrain the slope of the first item to a value of 0.98606, right-click in this field, select **Fix** from the pop-up menu, and enter the value as shown below.



Click **OK** after entering the value to return to the **Item Parameter Constraints** dialog box where the value of the constraint imposed is now shown in red.



Do the same for all the items for which constraints are to be imposed. The completed **Item Parameter Constraints** dialog box looks like this:



Click **OK** to return to the **Unidimensional** dialog box and proceed with setting up any other selections prior to running the generated syntax file.

3. Output

The tables reporting model parameter estimates for the binary and non-binary items in the output file clearly shows the constraints imposed.

3PL Model Item Parameter Estimates for Group 1, logit: $a\theta + c$ or $a(\theta - b)$

Item	Label		*a*	*s.e.*		*c*	*s.e.*	*b*	*s.e.*		logit *g*	*s.e.*	*g*	*s.e.*
1	C1		0.99	-----		0.77	-----	-0.78	-----		-1.53	-----	0.18	-----
2	C2		1.70	-----		2.12	-----	-1.25	-----		-1.60	-----	0.17	-----
3	C3		1.15	-----		-0.82	-----	0.72	-----		-2.53	-----	0.07	-----
4	C4		2.28	-----		1.03	-----	-0.45	-----		-2.25	-----	0.10	-----
5	C5		0.73	-----		0.07	-----	-0.09	-----		-1.43	-----	0.19	-----
6	C6	3	2.94	0.38	2	-1.20	0.24	0.41	0.05	1	-3.03	0.45	0.05	0.02
8	C8	6	1.64	0.27	5	0.54	0.25	-0.33	0.19	4	-1.21	0.45	0.23	0.08
9	C9	9	3.44	0.50	8	-2.62	0.41	0.76	0.05	7	-3.36	0.39	0.03	0.01
12	C12	15	1.04	0.15	14	0.37	0.19	-0.35	0.21	13	-1.89	0.63	0.13	0.07
13	C13	18	1.46	0.18	17	0.84	0.18	-0.58	0.16	16	-1.83	0.57	0.14	0.07

Graded Model Item Parameter Estimates, logit: $a\theta + c$

Item	Label		*a*	*s.e.*		c_1	*s.e.*		c_2	*s.e.*	c_3	*s.e.*
7	C7		2.41	-----		-0.82	-----		-3.09	-----	-4.68	-----
10	C10		2.60	-----		-2.12	-----		-4.06	-----		
11	C11	12	1.85	0.13	10	-0.29	0.09	11	-3.00	0.16		
14	C14	21	0.78	0.08	19	2.31	0.12	20	-0.07	0.07		

Graded Model Item Parameter Estimates for Group 1, logit: $a(\theta - b)$

Item	Label		*a*	*s.e.*	b_1	*s.e.*	b_2	*s.e.*	b_3	*s.e.*
7	C7		2.41	-----	0.34	-----	1.28	-----	1.94	-----
10	C10		2.60	-----	0.82	-----	1.56	-----		
11	C11	12	1.85	0.13	0.16	0.05	1.62	0.09		
14	C14	21	0.78	0.08	-2.96	0.30	0.10	0.09		

4. Linking/Equating Issues

To calibrate new items using fixed anchor item parameters that have been previously calibrated, one must determine whether the old and new groups are randomly equivalent. If the current sample is not a randomly equivalent group of individuals (i.e., from the same population) as the old, the population /prior mean and variance should be set to free estimated. This allows the maximum marginal likelihood procedure to appropriately characterize the population distribution, while simultaneously estimating the item parameters. To see the effect of population distribution on parameter linking, consider the logit of 2PL for the original item parameters:

$$a\theta + c,$$

with $\theta \sim N(0,1)$. These a and c values become fixed anchor values.

The new fixed parameter calibration, assuming freed population mean/SD, and logit of 2PL model for new items is:

$$\tilde{a}\tilde{\theta} + \tilde{c}.$$

Note that $\tilde{\theta} \sim N(\mu, \sigma)$ with the μ and σ values estimated by IRTPRO™. In other words, we can also write $\tilde{\theta} = \sigma\theta + \mu$. Substitution leads to the following expression:

$$\tilde{a}(\sigma\theta + \mu) + \tilde{c},$$

that is

$$[\tilde{a}\sigma]\theta + [\tilde{a}\mu + \tilde{c}].$$

Phrased another way, μ and σ are essential for placing the estimated item parameters on the same scale as the anchor items.